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Choked gas flow through pipeline restrictions: an explicit formula for the inlet Mach number

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Abstract

A simple analytical formula is derived for the inlet Mach number of a pipeline restriction through which there is choked flow of gas. This formula, valid for all diameter ratios $\beta < 1.0$, permits the explicit calculation of choked flow parameters to an accuracy of $< 0.1\%$, thus offering simplicity in comparison with existing iterative procedures. The formula can be applied directly in the design of pressure relief systems.

Keywords: Choked gas flow; Pipeline restriction; Inlet Mach number; Pressure relief system; Safety valve; Control valve

1. Introduction

Pipeline restrictions such as safety valves, control valves, etc., are often modelled as nozzles with an appropriate discharge coefficient C_D (non-ideal flow). In engineering design calculations relating to the sizing of various restriction types for gas service, it may be necessary to calculate the Mach number at the inlet to the restriction since this quantity is required for calculations of pressure loss in the upstream pipework. This Mach number is dependent both on the ratio of specific heats γ and the diameter ratio β (diameter of restriction throat/inlet pipe diameter).

The case of isentropic flow through a (frictionless) nozzle has recently been considered in this regard and the following analytical formula was derived for the inlet Mach number [1],

$$Ma_1 = \frac{\beta^2}{1 + \sqrt{\frac{1}{15}(4\gamma + 3)(1 - \beta^4)}} \tag{1}$$

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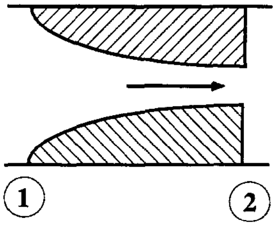

Fig. 1. Model of a pipeline restriction.

This Equation was shown to be valid for $\beta \leq 0.99$ and specific heat ratios in the range $1.0 \leq \gamma \leq 3.5$. However, this formula is no longer valid when there are irreversible losses in the flow due to friction and form drag. It is the purpose, therefore, of the present work to derive an equation similar to Eq. (1) but now for the case of restrictions with losses.

2. Analysis

The Mach number at any location in the nozzle of Fig. 1 is defined by:

$$Ma = u\sqrt{\frac{\tilde{M}}{\gamma Z R_o T_s}} \tag{2}$$

Making use of the equation of state for a non-ideal gas, i.e.

$$p_s v = Z R_o T_s / \tilde{M} \tag{3}$$

where the compressibility factor Z is assumed to be constant, and the equation of mass continuity:

$$\dot{M} = \rho_1 A_1 u_1 = \rho_2 A_2 u_2 \tag{4}$$

it is straightforward to show that the inlet and throat Mach numbers are related as follows:

$$Ma_1 = \beta^2 \eta \sqrt{\frac{T_{s1}}{T_{s2}}}\, Ma_2 \tag{5}$$

where

$$\eta = p_{s2}/p_{s1} \tag{6}$$

is the static pressure ratio.

Although not isentropic, the flow through the nozzle may be assumed to be adiabatic to good approximation, in which case the total (or stagnation) temperature is constant throughout the flowpath. The total temperature T_t is the temperature the gas would have if it were brought to rest isentropically and is related to the static temperature, T_s, by [2]:

$$T_t = T_s\left\{1 + \frac{1}{2}(\gamma - 1) Ma^2\right\} \tag{7}$$

Thus, the static temperatures at the nozzle inlet and throat are related by:

$$T_{s1}\left\{1+\frac{1}{2}(\gamma-1)Ma_1^2\right\}=T_{s2}\left\{1+\frac{1}{2}(\gamma-1)Ma_2^2\right\} \tag{8}$$

Now, the case of real interest here is that of choked flow, for which $Ma_2 = 1$. Then, substituting for T_{s1}/T_{s2} from Eq. (8) in Eq. (5) gives:

$$Ma_1=\beta^2\eta_c\sqrt{\frac{\gamma+1}{2\left\{1+\frac{1}{2}(\gamma-1)Ma_1^2\right\}}} \tag{9}$$

Where η_c is the static choke pressure ratio. Recalling a result from previous work [3], η_c is given by:

$$\eta_c=C_D r_{c,id}\left\{1+\frac{1}{2}(\gamma-1)Ma_1^2\right\}^{\frac{\gamma}{\gamma-1}} \tag{10}$$

where

$$r_{c,id}=p_c/p_{t1}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{11}$$

is the familiar static-to-total choke pressure ratio. Eqs. (9)–(11) may now be combined and rearranged into the following convenient form:

$$1+\frac{1}{2}(\gamma-1)\beta^4C_D^2\left(\frac{Ma_1^2}{\beta^4C_D^2}\right)=\frac{1}{2}(\gamma+1)\left(\frac{Ma_1^2}{\beta^4C_D^2}\right)^{\frac{\gamma-1}{\gamma+1}} \tag{12}$$

In the case $C_D = 1$ (lossless isentropic flow), this transcendental equation is identical to that solved previously [1] and for which Eq. (1) is the solution. Therefore, the solution of Eq. (12) for $C_D \neq 1$ is also given by Eq. (1) but now with $\beta^2 C_D$ replacing β^2, i.e.

$$Ma_1=\frac{\beta^2C_D}{1+\sqrt{\frac{1}{15}(4\gamma+3)\left(1-\beta^4C_D^2\right)}} \tag{13}$$

and has validity for $\beta\sqrt{C_D} \leq 0.99$ and $1.0 \leq \gamma \leq 3.5$, ranges which cover most cases of practical interest.The utility of this result is that Ma_1 can be calculated explicitly knowing γ, β and C_D and thus avoids the usual iterative procedures.

Eq. (13) may now be inserted in Eq. (10) in order to calculate the static choke pressure ratio η_c to an accuracy better than 0.1% (compared with the precise numerical solution performed on a computer), in accordance with the analysis presented previously [1]. The range of γ may even be extended above 3.5 while still maintaining high accuracy, e.g. at $\gamma = 5$, the maximum error over the full range of β is 0.53%.

As regards the choking mass flowrate $\dot{M}_c$, this is given by:

$$\dot{M}_c=C_D\dot{M}_{c,id} \tag{14}$$

where [2]

$$\dot{M}_{c,id} = A_2 p_{t1} \sqrt{\frac{\gamma \tilde{M}}{Z R_o T_{t1}}} \, r_{c,id}^{\frac{\gamma+1}{2\gamma}} \quad (15)$$

and $r_{c,id}$ is given by Eq. (11).

Although the errors in values of η_c calculated from Eq. (10) are less than 0.1%, at low Mach numbers the errors in values of Ma_1 calculated from Eq. (13) can be greater than this. This has no significant influence on η_c, however, since in this low Mach number range the term $\frac{1}{2}(\gamma - 1)Ma_1^2$ in Eq. (10) is very small compared with 1.

Since it is of interest to maintain the highest precision in Ma_1 for purposes of inlet pipe pressure drop calculations, it is recommended that an 'updated' value of Ma_1 is obtained from the equation

$$Ma_1^2 = \frac{1}{\gamma - 1}\left\{\sqrt{1 + (\gamma - 1)(\gamma + 1)\beta^4 \eta_c^2} - 1\right\} \quad (16)$$

after η_c has been determined from Eqs. (10) and (13).

The derivation of Eq. (16) follows from the observation that Eq. (9) can be expressed as a quadratic equation as follows:

$$z^2 - z - \frac{1}{4}(\gamma - 1)(\gamma + 1)\beta^4 \eta_c^2 = 0 \quad (17)$$

where

$$z = 1 + \frac{1}{2}(\gamma - 1) Ma_1^2 \quad (18)$$

This has the solution

$$z = \frac{1}{2}\left\{1 + \sqrt{1 + (\gamma - 1)(\gamma + 1)\beta^4 \eta_c^2}\right\} \quad (19)$$

from which Eq. (16) follows directly, thus permitting Ma_1 to be determined to within the same accuracy as η_c.

3. Choice of exponent γ

The use of a constant compressibility factor Z to introduce non-ideality into gas flow equations is very convenient and therefore widespread. However, this approximation has its limitations since the gas continues to obey the ideal gas law

$$pv = R'T \quad (20)$$

where

$$R' = ZR \quad (21)$$

Thus, all the ideal gas flow formulae continue to apply with the modification that R is replaced by ZR providing that the enthalpy of the gas continues to be reasonably approximated by the ideal gas formula, $h = c_p T$, where c_p is a constant. Nevertheless, this modification adds flexibility to the ideal gas flow formulae, permitting their use where Z is a weakly varying function of pressure and temperature. This may include a range of supercritical conditions but excludes conditions close to the critical point where Z is a particularly strong function of pressure and temperature.

A difficulty that regularly arises in safety valve sizing calculations is the choice of γ for process conditions under which γ differs significantly from the published values that pertain at relatively low pressures and temperatures. In such cases, γ and Z should be chosen together since they have an interdependency.

In the absence of readily available physical properties for the gas under the conditions of interest, the generalised charts of Reynolds [4] for compressibility factor, enthalpy and entropy can be very useful. These charts are presented in terms of reduced pressures given by $p_r = p_s/p_c$ and $T_r = T_s/T_c$ respectively.

One approach, which retains the simplicity of hand-calculations, is to choose reduced pressure–temperature pairs (p_{r1}, T_{r1}) and (p_{r2}, T_{r2}) which lie on a line of constant enthalpy (generalised enthalpy chart) and which span the estimated conditions across the restriction (or safety valve). Then, the corresponding compressibility factors Z_1 and Z_2 may be read from the generalised compressibility charts and used to determine γ from:

$$\gamma = \frac{\ln(p_{r1} p_{r2})}{\ln\left(\dfrac{Z_2 T_{r2} p_{r1}}{Z_1 T_{r1} p_{r2}}\right)} \tag{22}$$

This equation easily follows from:

$$p_{s1} v_1^{\gamma} = p_{s2} v_2^{\gamma} \tag{23}$$

and Eqs. (20) and (21). The value of γ found in this way may then be used in the foregoing analysis to calculate the Mach number at the restriction inlet.

A constant value of the compressibility factor Z is required for restriction flow capacity/sizing calculations that involve Eqs. (14) and (15). Providing that the variation of Z is approximately linear with pressure and temperature and the difference between Z_1 and Z_2 is not too great, it is reasonable to take the mean value, i.e.

$$Z = \frac{1}{2}(Z_1 + Z_2) \tag{24}$$

From the point of view of safety valve sizing, it is conservative to choose the larger of Z_1 and Z_2. In cases where such an approximation is not justified, e.g. near the critical point, then the only recourse is to an accurate equation of state and its incorporation in a flow model for the restriction (see Ref. [5]). Until this matter is considered further, the results of the present work should not be used in problems where a reasonable estimate for Z (and γ) cannot be achieved by the method described above.

4. Conclusions

An explicit formula (Eq. (13)) for determining the Mach number at the inlet of pipeline restrictions has been derived and is valid over the following ranges of diameter and specific heat ratios: $\beta\sqrt{C_D} \leq 0.99$; $1.0 \leq \gamma \leq 3.5$. At very low Mach numbers, this formula may exhibit maximum errors of a few percent; thus in order to maintain high precision throughout the whole Mach number range it is recommended that Eq. (16) is used to provide an 'updated' value of Ma_1, where η_c in Eq. (16) is calculated from Eqs. (10) and (13). The maximum error thus incurred is less than 0.1% for both the inlet Mach number and the choke pressure ratio when compared with the precise numerical solution. This simple and accurate formula is convenient for engineering calculations and is recommended for use in the design of emergency pressure relief systems involving safety valves.

5. Notation

A Area (m^2)
C_D Discharge coefficient
c_p Specific heat at constant pressure ($J\,kg^{-1}\,K^{-1}$)
D Diameter (m)
h Enthalpy ($J\,kg^{-1}$)
$\dot{M}$ Mass flowrate ($kg\,s^{-1}$)
$\tilde{M}$ Molar mass or molecular weight ($kg\,kmol^{-1}$)
Ma Mach number
p Pressure (Pa)
p_c Critical pressure (Pa)
P_r Reduced pressure
r Static-to-total pressure ratio
R Individual gas constant
R_o Universal gas constant ($8314\,J\,kmol^{-1}\,K^{-1}$)
R' Modified individual gas constant
T Temperature (K)
T_c Critical temperature (K)
T_r Reduced temperature
u Velocity ($m\,s^{-1}$)
v Specific volume ($m^3\,kg^{-1}$)
Z Compressibility factor for non-ideal gas
z Parameter defined by Eq. (17)
β Diameter ratio D_2/D_1 of restriction
η Static pressure ratio p_{s2}/p_{s1}
γ Ratio of specific heats (c_p/c_v)

5.1. Subscripts

1 Inlet of restriction
2 Throat of restriction

c Choke value
id Ideal flow
s Static
t Total

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